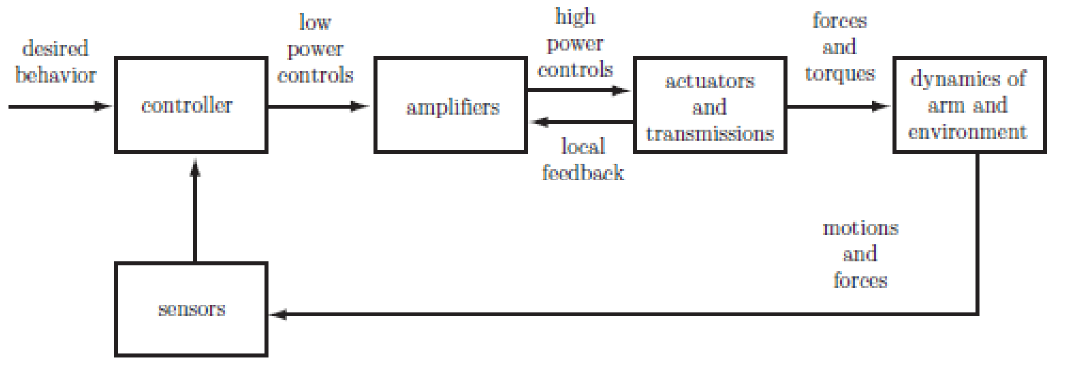
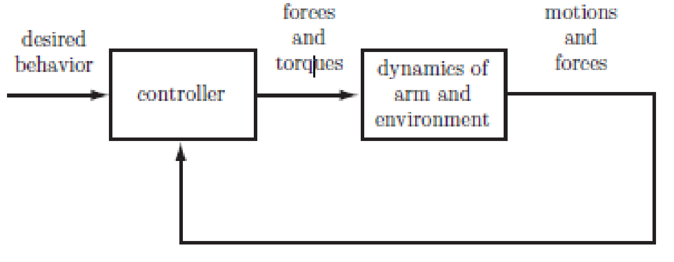
Robot Control

Overview

- Typical control system
  - Types: motion control / force control
  - Actual block diagram



  - Simplified block diagram



Error Dynamics

- Joint error

  $\theta_e(t) = \theta_d(t) - \theta(t)$ desired joint position is $\theta_d(t)$ actual joint position is $\theta(t)$

- Error Dynamics
  - Differential equations for the joint error (ex) $\ddot{\theta}_e(t) + \frac{b}{m}\dot{\theta}_e(t) + \frac{k}{m}\theta_e(t) = 0$

- Error Response

  $\theta_e(t), t > 0$ for the initial conditions

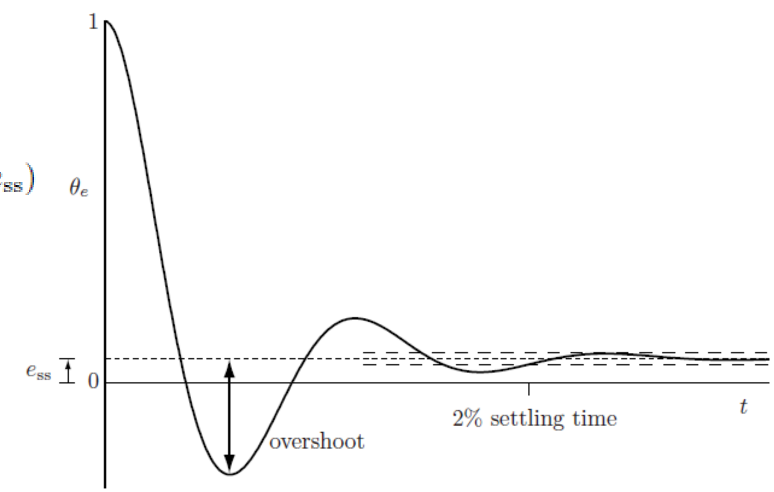
  $$\theta_e(0) = 1 \quad \dot{\theta}_e(0) = \ddot{\theta}_e(0) = \cdots = 0$$

  1) Steady state error e_{ss}
     asymptotic error as $t \rightarrow \infty$

  2) (2%) Settling time
     the first time T such that $|\theta_e(t) - e_{ss}| \leq 0.02(\theta_e(0) - e_{ss})$ for all $t \geq T$

  3) Overshoot

  $$\text{overshoot} = \left|\frac{\theta_{e,\min} - e_{ss}}{\theta_e(0) - e_{ss}}\right| \times 100\%,$$



Error Dynamics

- Linear error dynamics
  - Linear ordinary differential equations

$$\boxed{a_p\theta_e^{(p)} + a_{p-1}\theta_e^{(p-1)} + \cdots + a_2\ddot{\theta}_e + a_1\dot{\theta}_e + a_0\theta_e = c.}$$

$c = 0$: homogeneous
$c \neq 0$: nonhomogeneous

$$\begin{aligned}\theta_e^{(p)} &= -\frac{1}{a_p}(a_{p-1}\theta_e^{(p-1)} + \cdots + a_2\ddot{\theta}_e + a_1\dot{\theta}_e + a_0\theta_e) \\ &= -a'_{p-1}\theta_e^{(p-1)} - \cdots - a'_2\ddot{\theta}_e - a'_1\dot{\theta}_e - a'_0\theta_e.\end{aligned}$$

  - Matrix form
    - State vector $x = (x_1, \ldots, x_p)$

$$\begin{aligned} x_1 &= \theta_e, \\ x_2 &= \dot{x}_1 = \dot{\theta}_e, \\ \vdots &\quad \vdots \\ x_p &= \dot{x}_{p-1} = \theta_e^{(p-1)} \\ \dot{x}_p &= -a'_0 x_1 - a'_1 x_2 - \cdots - a'_{p-1} x_p. \end{aligned}$$

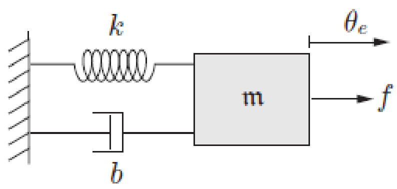
    - State equation

$$\boxed{\dot{x}(t) = Ax(t)}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ -a'_0 & -a'_1 & -a'_2 & \cdots & -a'_{p-2} & -a'_{p-1} \end{bmatrix}$$

Error Dynamics

- mass-spring-damper system



$$m\ddot{\theta}_e + b\dot{\theta}_e + k\theta_e = f.$$

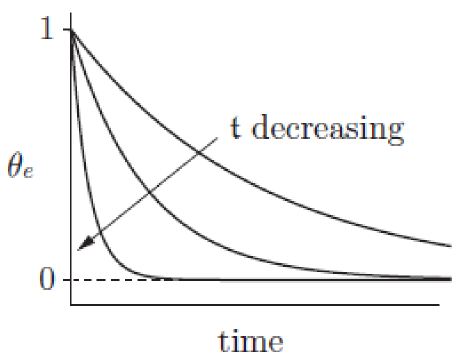
- First-order error dynamics (m=0, f=0)

$$\dot{\theta}_e(t) + \frac{k}{b}\theta_e(t) = 0$$

$$\Rightarrow \boxed{\dot{\theta}_e(t) + \frac{1}{\mathrm{t}}\theta_e(t) = 0} \qquad \theta_e(t) = e^{-t/\mathrm{t}}\theta_e(0)$$

t: time constant

decayed to 37% of initial value



✓ 2% settling time ≒ 4 t

$$\frac{\theta_e(t)}{\theta_e(0)} = 0.02 = e^{-t/\mathrm{t}}$$

$$\ln 0.02 = -t/\mathrm{t} \rightarrow t = 3.91\mathrm{t},$$

Error Dynamics

- Second-order error dynamics (f=0)

$$\ddot{\theta}_e(t) + \frac{b}{m}\dot{\theta}_e(t) + \frac{k}{m}\theta_e(t) = 0 \qquad \omega_n = \sqrt{k/m},\ \zeta = b/2\sqrt{km}$$

$$\Rightarrow \boxed{\ddot{\theta}_e(t) + 2\zeta\omega_n\dot{\theta}_e(t) + \omega_n^2\theta_e(t) = 0,}$$

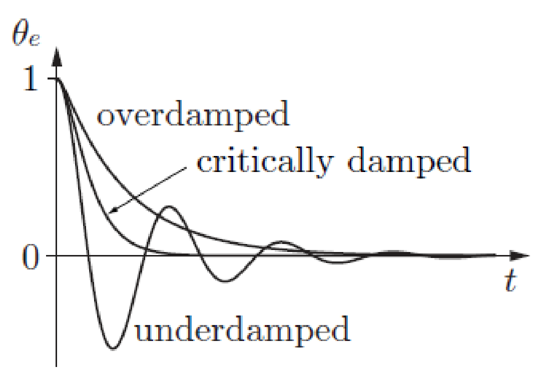
ω_n: natural frequency, ζ : damping ratio

$$\Rightarrow s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$ (characteristic equation)

$$s_1 = -\zeta\omega_n + \omega_n\sqrt{\zeta^2 - 1} \quad \text{and} \quad s_2 = -\zeta\omega_n - \omega_n\sqrt{\zeta^2 - 1}.$$

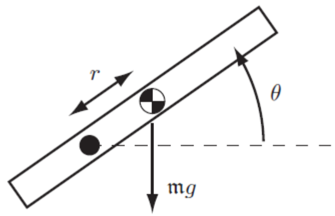
$\Rightarrow$ Stable condition: $\zeta\omega_n > 0$

1) Overdamped ($\varsigma > 1$) $\qquad \theta_e(t) = c_1 e^{s_1 t} + c_2 e^{s_2 t}$
2) Critically damped ($\varsigma = 1$) $\qquad \theta_e(t) = (c_1 + c_2 t)e^{-\omega_n t}$
3) Underdamped ($\varsigma < 1$)) $\qquad \theta_e(t) = (c_1 \cos\omega_d t + c_2 \sin\omega_d t)\, e^{-\zeta\omega_n t}$



Motion Control of a Single Joint

- Arm dynamics
  - Single motor attached to a single link



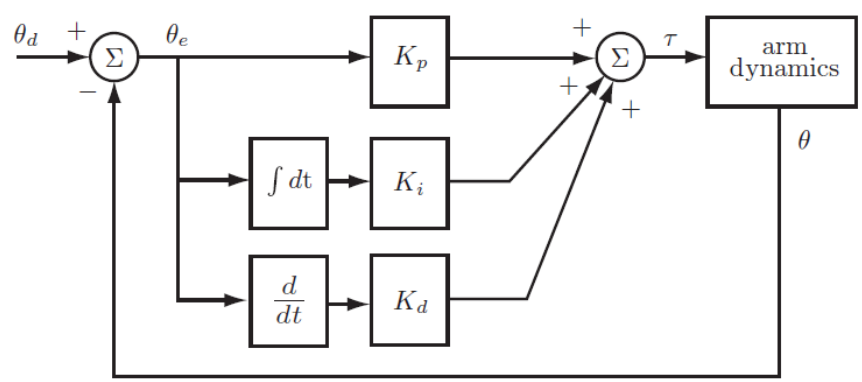
$$\boxed{\tau = M\ddot{\theta} + \mathrm{m}gr\cos\theta + b\dot{\theta}}$$
$$= M\ddot{\theta} + h(\theta, \dot{\theta})$$

τ: motor's torque
θ: angle of link
M: inertia of link
m: mass of link
r: distance from axis to center of mass
g: gravitational accel.
b: viscous coeff.

- Feedback control: PID control

$$\boxed{\tau = K_p\theta_e + K_i\int \theta_e(\mathrm{t})d\mathrm{t} + K_d\dot{\theta}_e}$$

$$\theta_e = \theta_d - \theta$$



Motion Control of a Single Joint

- PD Control (g=0)
  - System dynamics ($\dot{\theta}_d = \ddot{\theta}_d = 0$)

$$M\ddot{\theta} + b\dot{\theta} = K_p(\theta_d - \theta) + K_d(\dot{\theta}_d - \dot{\theta})$$

$$\Rightarrow \boxed{\ddot{\theta}_e + \frac{b + K_d}{M}\dot{\theta}_e + \frac{K_p}{M}\theta_e = 0}$$

$$\Rightarrow \ddot{\theta}_e + 2\zeta\omega_n\dot{\theta}_e + \omega_n^2\theta_e = 0 \qquad \zeta = \frac{b + K_d}{2\sqrt{K_p M}} \quad \text{and} \quad \omega_n = \sqrt{\frac{K_p}{M}}$$

  - Stable condition

$$b + K_d > 0 \qquad K_p > 0$$

  - Critical damping: $\varsigma = 1$

Increasing K_p ⇒ decreasing ζ ⇒ underdamped
Increasing K_d ⇒ increasing ζ ⇒ overdamped / reduce settling time

  - Steady-state error = 0 ($t \to \infty \Rightarrow \dot{\theta}_e = \ddot{\theta}_e = 0 \Rightarrow \theta_e = 0$)

Motion Control of a Single Joint

- PD Control (g>0)
  - System dynamics

$$M\ddot{\theta}_e + (b + K_d)\dot{\theta}_e + K_p\theta_e = \mathfrak{m}gr\cos\theta$$

  - Steady-state error

$$\theta_e = \frac{mgr\cos\theta}{K_p}$$

- PID Control (g>0)
  - System dynamics

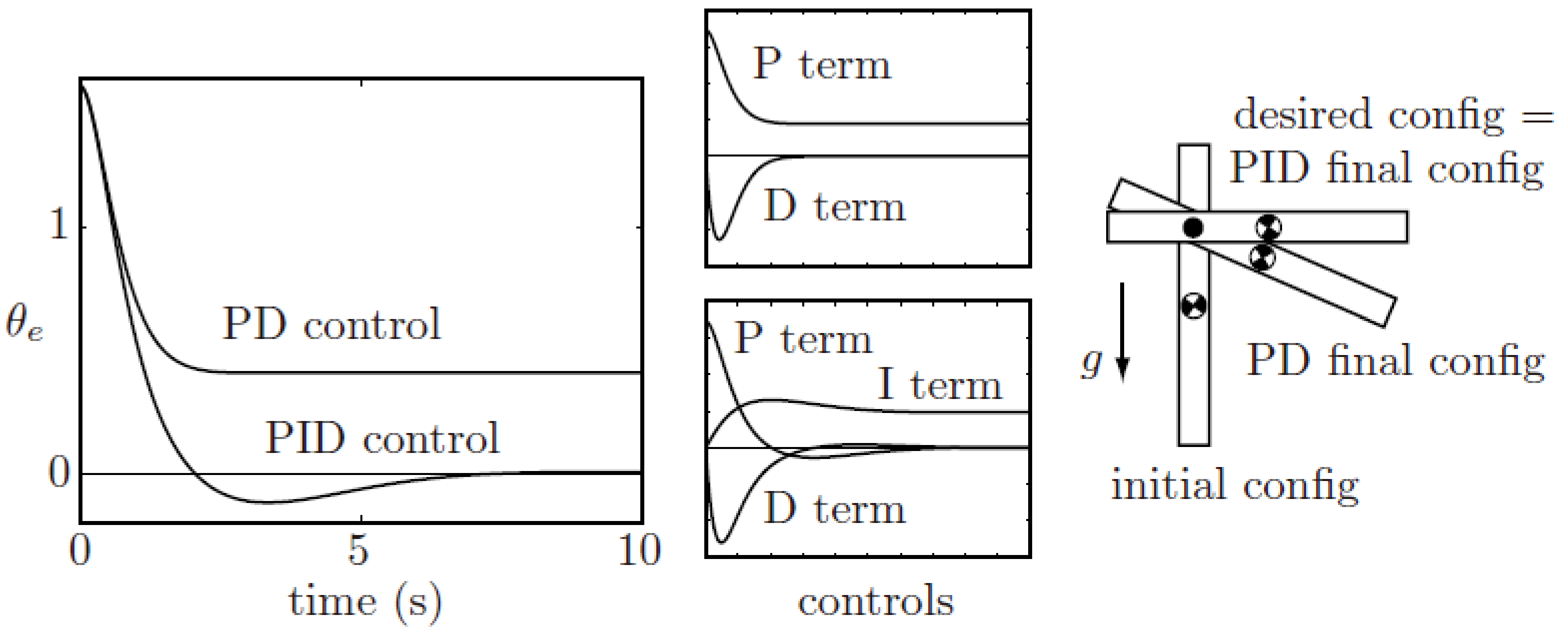
$$M\ddot{\theta}_e + (b + K_d)\dot{\theta}_e + K_p\theta_e + K_i\int\theta_e(\mathrm{t})d\mathrm{t} = \tau_{\text{dist}}$$

$$\Rightarrow \boxed{M\theta_e^{(3)} + (b + K_d)\ddot{\theta}_e + K_p\dot{\theta}_e + K_i\theta_e = \dot{\tau}_{\text{dist}}}$$

  - Steady state error = 0 (if $\dot{\tau}_{\text{dist}}$=0)

Motion Control of a Single Joint

- PD vs. PID



Motion Control of a Single Joint

- Pseudocode for PID control

```
time = 0                          // dt = servo cycle time
eint = 0                          // error integral
qprev = senseAngle                // initial joint angle q
loop
  [qd,qdotd] = trajectory(time) // from trajectory generator

  q = senseAngle                  // sense actual joint angle
  qdot = (q - qprev)/dt           // simple velocity calculation
  qprev = q

  e = qd - q
  edot = qdotd - qdot
  eint = eint + e*dt

  tau = Kp*e + Kd*edot + Ki*eint
  commandTorque(tau)

  time = time + dt
end loop
```

☞ Arm dynamics 고려하지 않음

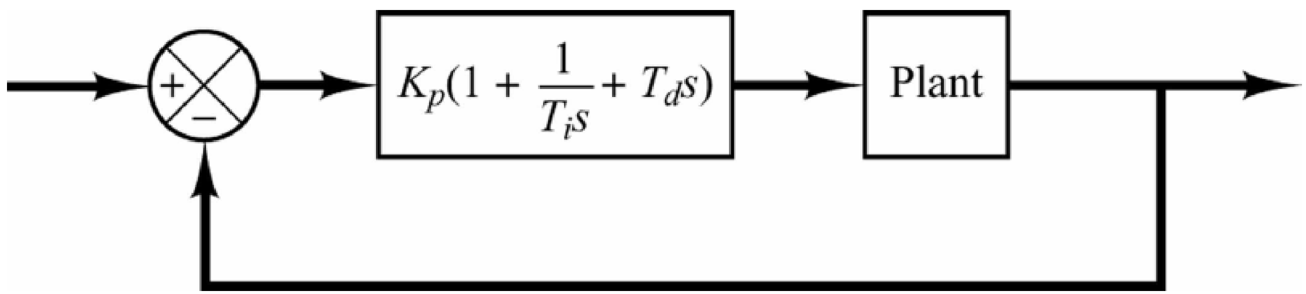
PID Tuning

- P, I, D Gain 의 관계

	Overshoot	Settling Time	Steady State Error
Kp	Increase	Small change	Decrease
Kd	Decrease	Decrease	None
Ki	Increase	Increase	Eliminate

PID Tuning

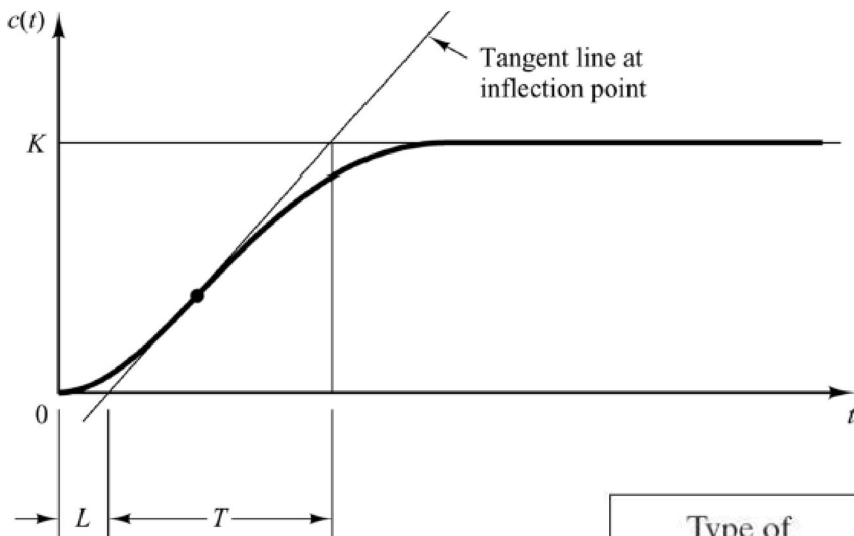
- Ziegler-Nichols Method
  - Plant 에 대한 step response (transient state) 로 부터 Kp, Kd, Ki 를 설정하는 실험적 방법



$$\frac{U(s)}{E(s)} = G_{PID}(s) = K_P + K_I \frac{1}{s} + K_D s = K_P \left(1 + \frac{1}{T_I s} + T_D s \right)$$

PID Tuning

- Ziegler-Nichols Method-Case 1
  - S-shaped step response

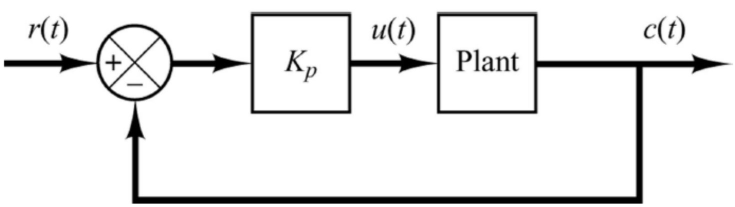


Type of Controller	K_p	T_i	T_d
P	$\frac{T}{L}$	∞	0
PI	$0.9\frac{T}{L}$	$\frac{L}{0.3}$	0
PID	$1.2\frac{T}{L}$	$2L$	$0.5L$

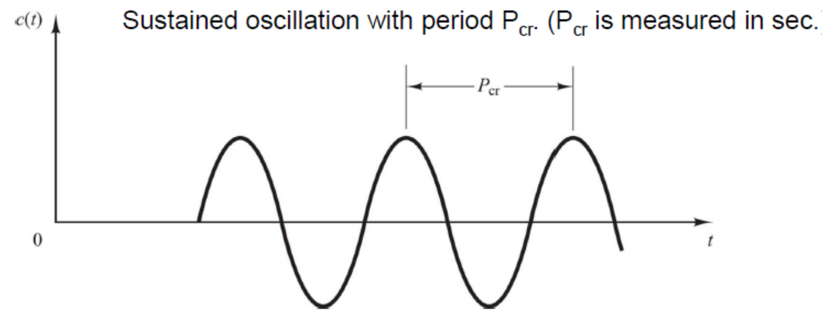
PID Tuning

- Ziegler-Nichols Method-Case 2

- Start with Closed-loop system with a proportional controller.
- Begin with a low value of gain, K_P
- Potential of this method to go unstable or cause damage.



- Begin with a low/zero value of gain K_P
- Increase until a steady-state oscillation occurs, note this gain as K_{cr}



Type of Controller	K_p	T_i	T_d
P	$0.5K_{cr}$	∞	0
PI	$0.45K_{cr}$	$\frac{1}{1.2} P_{cr}$	0
PID	$0.6K_{cr}$	$0.5P_{cr}$	$0.125P_{cr}$

Motion Control of a Single Joint

- Nonlinear arm dynamics

$$\tau = M(\theta)\ddot{\theta} + h(\theta, \dot{\theta})$$

- Computed torque

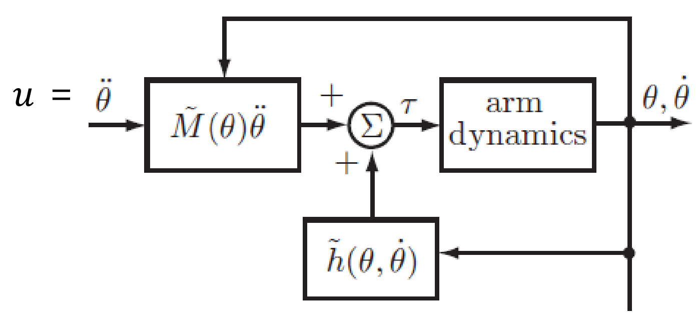
$$\tau = \tilde{M}(\theta)\ddot{\theta} + \tilde{h}(\theta, \dot{\theta})$$

Ideal case : $\tilde{M}(\theta) = M(\theta), \tilde{h}(\theta, \dot{\theta}) = h(\theta, \dot{\theta})$

- Feedback linearization
  - Feedback 을 통하여 비선형 시스템을 선형시스템으로 전환
  - Feedback 에 dynamics 계산 과정 포함
  - Dynamics 계산 오차가 큰 경우 성능저하

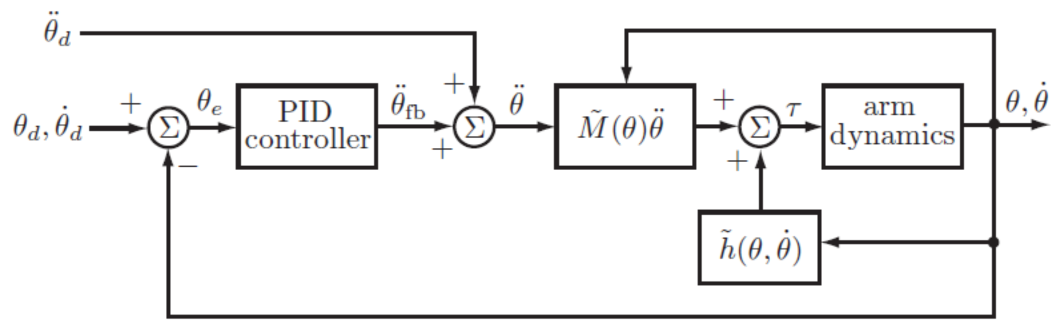
$$\begin{aligned} \tau &= M(\theta)\ddot{\theta} + h(\theta, \dot{\theta}) \\ &= \tilde{M}(\theta)u + \tilde{h}(\theta, \dot{\theta}) \end{aligned}$$

Ideal case ⇨ $\ddot{\theta} = u$ Linear system



Motion Control of a Single Joint

- Computed torque control
  - PID control + feedback linearization
    - Feedback linearization 을 통하여 비선형 arm dynamics 을 선형화 (linearization)
    - 선형 시스템과 같이 PID control 쉽게 적용



  - Control law

$$\ddot{\theta}_e + K_d\dot{\theta}_e + K_p\theta_e + K_i\int \theta_e(\mathrm{t})d\mathrm{t} = c \qquad \ddot{\theta} = \ddot{\theta}_d - \ddot{\theta}_e$$

$$\Rightarrow \ddot{\theta} = \ddot{\theta}_d + K_d\dot{\theta}_e + K_p\theta_e + K_i\int \theta_e(\mathrm{t})d\mathrm{t}$$

$$\Rightarrow \boxed{\tau = \tilde{M}(\theta)\left(\ddot{\theta}_d + K_p\theta_e + K_i\int \theta_e(\mathrm{t})d\mathrm{t} + K_d\dot{\theta}_e\right) + \tilde{h}(\theta, \dot{\theta})}$$

Motion Control of a Single Joint

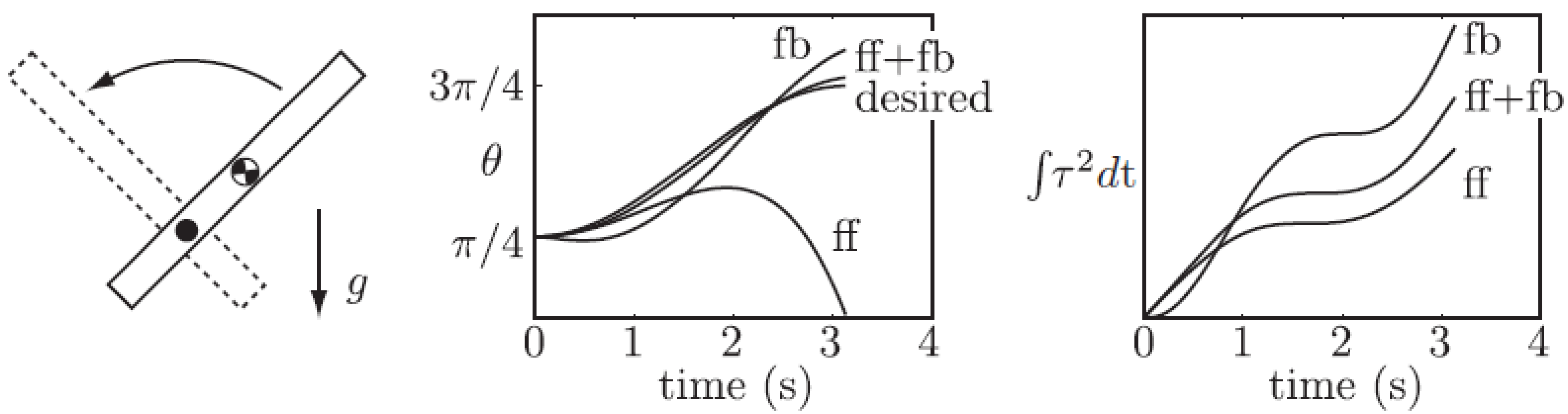
- Psedocode for computed torque control

```
time = 0                                    // dt = cycle time
eint = 0                                    // error integral
qprev = senseAngle                          // initial joint angle q
loop
  [qd,qdotd,qdotdotd] = trajectory(time)    // from trajectory generator

  q = senseAngle                            // sense actual joint angle
  qdot = (q - qprev)/dt                     // simple velocity calculation
  qprev = q

  e = qd - q
  edot = qdotd - qdot
  eint = eint + e*dt

  tau = Mtilde(q)*(qdotdotd+Kp*e+Kd*edot+Ki*eint) + htilde(q,qdot)
  commandTorque(tau)

  time = time + dt
end loop
```

☞ Arm dynamics 고려함

Motion Control of a Single Joint

- Performance



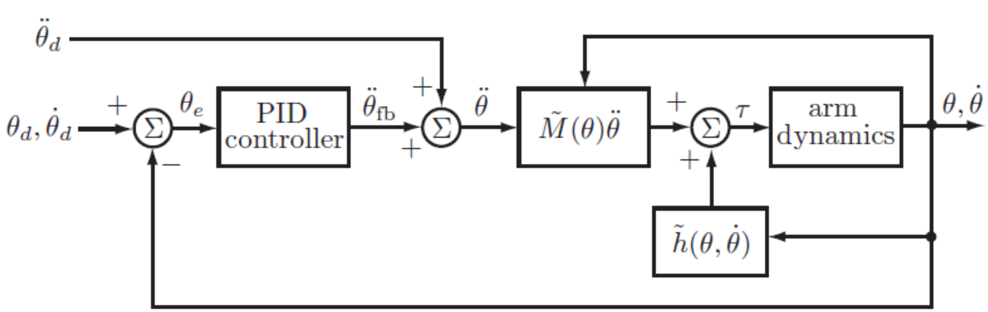
Multi-Joint Control

1. Decentralized control
   - Arm dynamics 가 decoupling 된 경우 (joint 간 상호영향이 없음)

     (ex) $M(\theta)$: diagonal matrix

     Cartesian type manipulator case, Geared motor case
   - Independent joint control: 각 joint 별 제어
     - PID control or computed torque control

2. Centralized control
   - Arm dynamics 가 decoupling 되지 않은 경우
   - Generalized computed torque control



θ, θ_d, τ: n x 1 vector

$\tilde{M}(\theta)$: n x n matrix

K_p, K_I, K_D : n x n matrix

$$\tau = \tilde{M}(\theta)\left(\ddot{\theta}_d + K_p\theta_e + K_i\int\theta_e(\mathrm{t})d\mathrm{t} + K_d\dot{\theta}_e\right) + \tilde{h}(\theta, \dot{\theta})$$