2. GAs: How Do They Work ?

Step 1. Population 초기화

$v_i \ (i = 1, \cdots, pop\_size)$

* random generation

Step 2. Reproduction (or Selection)

<Roulette Wheel 만들기>

2.1 각 chromosome $v_i \ (i = 1, \cdots, pop\_size)$ 에 대한 fitness $eval(v_i)$ 계산

2.2 population 의 total fitness 계산

$$F = \sum_{i=1}^{pop\_size} eval(v_i)$$

2.3 각 chromosome $v_i \ (i = 1, \cdots, pop\_size)$ 에 대한 선택확률 p_i (probability of selection) 계산

$$p_i = eval(v_i) / F$$

2.4 각 chromosome $v_i \ (i = 1, \cdots, pop\_size)$ 에 대한 누적확률 q_i (cumulative probability) 계산

$$q_i = \sum_{j=1}^{i} p_j$$

<Roulette Wheel 돌려 선택하기>

2.5 다음을 pop_size 회 반복

(1) Random number $r \in [0,1]$ 생성

(2) $q_{i-1} < r \le q_i \quad (i = 2, \cdots, pop\_size)$ 이면 i 번째 chromosome v_i 선택.

단, $r < q_1$ 이면 v_1 선택

=> 새로운 population 생성

Step 3. Crossover

3.1 Population 의 모든 chromosome 에 대하여, 다음 반복

(1) Random number $r \in [0,1]$ 생성

(2) $r < p_c$ 이면, (단 p_c : probability of crossover , 주어진 parameter)
crossover 대상 chromosome 으로 선택.

3.2 crossover 대상 chromosome 을 pair 로 묶는다.

3.3 각 pair 에 대하여,

(1) Crossing point 의 위치 결정
: random integer $pos \in [1, m-1]$ 생성. (단 m 은 chromosome 길이)

(2) Crossover

$$(b_1 b_2 \cdots b_{pos} b_{pos+1} \cdots b_m) \quad \text{and}$$
$$(c_1 c_2 \cdots c_{pos} c_{pos+1} \cdots c_m)$$
$$\Rightarrow$$
$$(b_1 b_2 \cdots b_{pos} c_{pos+1} \cdots c_m) \quad \text{and}$$
$$(c_1 c_2 \cdots c_{pos} b_{pos+1} \cdots b_m)$$

Step 4. Mutation

모든 chromosome 의 각 gene (bit) 에 대하여 다음 반복.

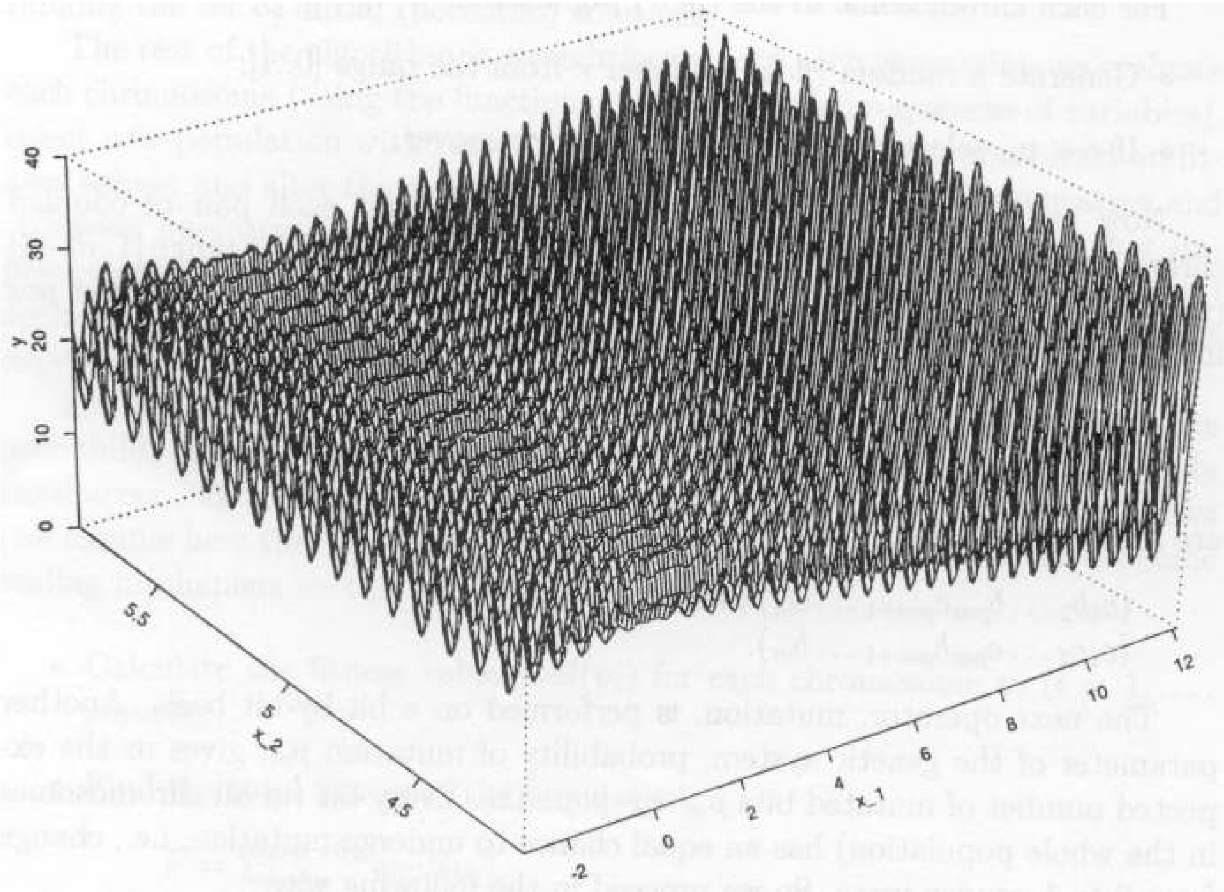
(1) Random number $r \in [0,1]$ 생성

(2) $r < p_m$ 이면, (단 p_m : probability of mutation , 주어진 parameter)
해당 bit mutate
0 -> 1, 1->0

Step 5. (Stop Condition 만족 못하면) Go to Step 2

(Ex) Maximize $f(x_1, x_2) = 21.5 + x_1 \sin(4\pi x_1) + x_2 \sin(20\pi x_2)$

where $-3.0 \le x_1 \le 12.1$, $4.1 \le x_2 \le 5.8$



● Representation

소수이하 4자리 표시시

$$x_1 \quad : \quad 2^{17} < 151000 \le 2^{18}$$
$$x_2 \quad : \quad 2^{14} < 17000 \le 2^{15}$$

∴ x_1 18 bit, x_2 15 bit : 총 33 bit -> 1 chromosome 구성

(ex) chromosome (010001001011010000111110010100010)

$$x_1 = -3.0 + decimal(010001001011010000_2) \cdot \frac{12.1-(-3.0)}{2^{18}-1}$$
$$= 1.052426$$
$$x_2 = 4.1 + decimal(111110010100010_2) \cdot \frac{5.8-4.1}{2^{15}-1}$$
$$= 5.755330$$

● Parameter

$$pop\_size = 20$$
$$p_c = 0.25$$
$$p_m = 0.01$$

Step 1. Population 초기화

∗ Random generation

$$
\begin{aligned}
\mathbf{v}_1 &= (100110100000001111111010011011111)\\
\mathbf{v}_2 &= (111000100100110111001010100011010)\\
\mathbf{v}_3 &= (000010000011001000001010111011101)\\
\mathbf{v}_4 &= (100011000101101001111000001110010)\\
\mathbf{v}_5 &= (000111011001010011010111111000101)\\
\mathbf{v}_6 &= (000101000010010101001010111111011)\\
\mathbf{v}_7 &= (001000100000110101111011011111011)\\
\mathbf{v}_8 &= (100001100001110100010110101100111)\\
\mathbf{v}_9 &= (010000000101100010110000001111100)\\
\mathbf{v}_{10} &= (000001111000110000011010000111011)\\
\mathbf{v}_{11} &= (011001111110110101100001101111000)\\
\mathbf{v}_{12} &= (110100010111101101000101010000000)\\
\mathbf{v}_{13} &= (111011111010001000110000001000110)\\
\mathbf{v}_{14} &= (010010011000001010100111100101001)\\
\mathbf{v}_{15} &= (111011101101110000100011111011110)\\
\mathbf{v}_{16} &= (110011110000011111100001101001011)\\
\mathbf{v}_{17} &= (011010111111001111010001101111101)\\
\mathbf{v}_{18} &= (011101000000001110100111110101101)\\
\mathbf{v}_{19} &= (000101010011111111110000110001100)\\
\mathbf{v}_{20} &= (101110010110011110011000101111110)
\end{aligned}
$$

Step 2. Reproduction (or Selection)

2.1 Fitness

$$\begin{aligned}
eval(\mathbf{v}_1) &= f(6.084492, 5.652242) = 26.019600\\
eval(\mathbf{v}_2) &= f(10.348434, 4.380264) = 7.580015\\
eval(\mathbf{v}_3) &= f(-2.516603, 4.390381) = 19.526329\\
eval(\mathbf{v}_4) &= f(5.278638, 5.593460) = 17.406725\\
eval(\mathbf{v}_5) &= f(-1.255173, 4.734458) = 25.341160\\
eval(\mathbf{v}_6) &= f(-1.811725, 4.391937) = 18.100417\\
eval(\mathbf{v}_7) &= f(-0.991471, 5.680258) = 16.020812\\
eval(\mathbf{v}_8) &= f(4.910618, 4.703018) = 17.959701\\
eval(\mathbf{v}_9) &= f(0.795406, 5.381472) = 16.127799\\
eval(\mathbf{v}_{10}) &= f(-2.554851, 4.793707) = 21.278435\\
eval(\mathbf{v}_{11}) &= f(3.130078, 4.996097) = 23.410669\\
eval(\mathbf{v}_{12}) &= f(9.356179, 4.239457) = 15.011619\\
eval(\mathbf{v}_{13}) &= f(11.134646, 5.378671) = 27.316702\\
eval(\mathbf{v}_{14}) &= f(1.335944, 5.151378) = 19.876294\\
eval(\mathbf{v}_{15}) &= f(11.089025, 5.054515) = 30.060205\\
eval(\mathbf{v}_{16}) &= f(9.211598, 4.993762) = 23.867227\\
eval(\mathbf{v}_{17}) &= f(3.367514, 4.571343) = 13.696165\\
eval(\mathbf{v}_{18}) &= f(3.843020, 5.158226) = 15.414128\\
eval(\mathbf{v}_{19}) &= f(-1.746635, 5.395584) = 20.095903\\
eval(\mathbf{v}_{20}) &= f(7.935998, 4.757338) = 13.666916
\end{aligned}$$

2.2 Total Fitness

$$F = \sum_{i=1}^{20} eval(\mathbf{v}_i) = 387.776822.$$

2.3 Probability of Selection

$$\begin{aligned}
p_1 &= eval(\mathbf{v}_1)/F = 0.067099 & p_2 &= eval(\mathbf{v}_2)/F = 0.019547\\
p_3 &= eval(\mathbf{v}_3)/F = 0.050355 & p_4 &= eval(\mathbf{v}_4)/F = 0.044889\\
p_5 &= eval(\mathbf{v}_5)/F = 0.065350 & p_6 &= eval(\mathbf{v}_6)/F = 0.046677\\
p_7 &= eval(\mathbf{v}_7)/F = 0.041315 & p_8 &= eval(\mathbf{v}_8)/F = 0.046315\\
p_9 &= eval(\mathbf{v}_9)/F = 0.041590 & p_{10} &= eval(\mathbf{v}_{10})/F = 0.054873\\
p_{11} &= eval(\mathbf{v}_{11})/F = 0.060372 & p_{12} &= eval(\mathbf{v}_{12})/F = 0.038712\\
p_{13} &= eval(\mathbf{v}_{13})/F = 0.070444 & p_{14} &= eval(\mathbf{v}_{14})/F = 0.051257\\
p_{15} &= eval(\mathbf{v}_{15})/F = 0.077519 & p_{16} &= eval(\mathbf{v}_{16})/F = 0.061549\\
p_{17} &= eval(\mathbf{v}_{17})/F = 0.035320 & p_{18} &= eval(\mathbf{v}_{18})/F = 0.039750\\
p_{19} &= eval(\mathbf{v}_{19})/F = 0.051823 & p_{20} &= eval(\mathbf{v}_{20})/F = 0.035244
\end{aligned}$$

2.4 Cumulative Probability

$q_1 = 0.067099$	$q_2 = 0.086647$	$q_3 = 0.137001$	$q_4 = 0.181890$
$q_5 = 0.247240$	$q_6 = 0.293917$	$q_7 = 0.335232$	$q_8 = 0.381546$
$q_9 = 0.423137$	$q_{10} = 0.478009$	$q_{11} = 0.538381$	$q_{12} = 0.577093$
$q_{13} = 0.647537$	$q_{14} = 0.698794$	$q_{15} = 0.776314$	$q_{16} = 0.837863$
$q_{17} = 0.873182$	$q_{18} = 0.912932$	$q_{19} = 0.964756$	$q_{20} = 1.000000$

2.5 (1) Random number generation $r \in [0, 1]$

0.513870	0.175741	0.308652	0.534534	0.947628
0.171736	0.702231	0.226431	0.494773	0.424720
0.703899	0.389647	0.277226	0.368071	0.983437
0.005398	0.765682	0.646473	0.767139	0.780237

(2) Selection

(ex) $r = 0.513870$ => $q_{10} < r \leq q_{11}$: v_{11} 선택

$r = 0.175741$ => $q_3 < r \leq q_4$: v_4 선택

......

$$
\begin{aligned}
v'_1 &= (011001111110110101100001101111000)\ (v_{11})\\
v'_2 &= (100011000101101001111000001110010)\ (v_4)\\
v'_3 &= (001000100000110101111011011111011)\ (v_7)\\
v'_4 &= (011001111110110101100001101111000)\ (v_{11})\\
v'_5 &= (000101010011111111110000110001100)\ (v_{19})\\
v'_6 &= (100011000101101001111000001110010)\ (v_4)\\
v'_7 &= (111011101101110000100011111011110)\ (v_{15})\\
v'_8 &= (000111011001010011010111111000101)\ (v_5)\\
v'_9 &= (011001111110110101100001101111000)\ (v_{11})\\
v'_{10} &= (000010000011001000001010111011101)\ (v_3)\\
v'_{11} &= (111011101101110000100011111011110)\ (v_{15})\\
v'_{12} &= (010000000101100010110000001111100)\ (v_9)\\
v'_{13} &= (000101000010010101001010111111011)\ (v_6)\\
v'_{14} &= (100001100001110100010110101100111)\ (v_8)\\
v'_{15} &= (101110010110011110011000101111110)\ (v_{20})\\
v'_{16} &= (100110100000001111111010011011111)\ (v_1)\\
v'_{17} &= (000001111000110000011010000111011)\ (v_{10})\\
v'_{18} &= (111011111010001000110000001000110)\ (v_{13})\\
v'_{19} &= (111011101101110000100011111011110)\ (v_{15})\\
v'_{20} &= (110011110000011111100001101001011)\ (v_{16})
\end{aligned}
$$

Step 3. Crossover

3.1 Chromosome 선택

(1) Random number generation : $r \in [0.1]$

0.822951	0.151932	0.625477	0.314685	0.346901
0.917204	0.519760	0.401154	0.606758	0.785402
0.031523	0.869921	0.166525	0.674520	0.758400
0.581893	0.389248	0.200232	0.355635	0.826927

(2) $r < p_c$ (0.25) 인 chromosome 선택 (기대치 = pop_size * p_c)

v_2', v_{11}', v_{13}' , v_{18}'

3.2 Crossover Pair 결정 : (v_2', v_{11}'), (v_{13}' , v_{18}')

3.3 Crossover

(1) Crossing Point 결정 : random number $pos \in [1, m-1]$

pos = 9,   pos = 20

(2) Crossover

$$v_2' = (100011000|101101001111000001110010)$$
$$v_{11}' = (111011101|101110000100011111011110)$$

=>
$$v_2'' = (100011000|101110000100011111011110)$$
$$v_{11}'' = (111011101|101101001111000001110010)$$

$$v_{13}' = (00010100001001010100|1010111111011)$$
$$v_{18}' = (11101111101000100011|0000001000110)$$

=>
$$v_{13}'' = (00010100001001010100|0000001000110)$$
$$v_{18}'' = (11101111101000100011|1010111111011).$$

(Current Population)

$$
\begin{aligned}
v'_1 &= (011001111110110101100001101111000)\\
\boxed{v''_2} &= (100011000101110000100011111011110)\\
v'_3 &= (001000100000110101111011011111011)\\
v'_4 &= (011001111110110101100001101111000)\\
v'_5 &= (000101010011111111110000110001100)\\
v'_6 &= (100011000101101001111000001110010)\\
v'_7 &= (111011101101110000100011111011110)\\
v'_8 &= (000111011001010011010111111000101)\\
v'_9 &= (011001111110110101100001101111000)\\
v'_{10} &= (000010000011001000001010111011101)\\
\boxed{v''_{11}} &= (111011101101101001111000001110010)\\
v'_{12} &= (010000000101100010110000001111100)\\
\boxed{v''_{13}} &= (000101000010010101000000001000110)\\
v'_{14} &= (100001100001110100010110101100111)\\
v'_{15} &= (101110010110011110011000101111110)\\
v'_{16} &= (100110100000001111111010011011111)\\
v'_{17} &= (000001111000110000011010000111011)\\
\boxed{v''_{18}} &= (111011111010001000111010111111011)\\
v'_{19} &= (111011101101110000100011111011110)\\
v'_{20} &= (110011110000011111100001101001011)
\end{aligned}
$$

Step 4. Mutation

(1) random nimber r ∈ [0,1] 을 Population 의 모든 bit 에 대하여 생성

(2) $r < p_m$ (0.01) 인 bit 를 선택하여 mutation

* 기대치 = m × pop_size × p_m = 33 × 20 × 0.01 = 6.6 (bit)

Bit position	Random number
112	0.000213
349	0.009945
418	0.008809
429	0.005425
602	0.002836

Bit position	Chromosome number	Bit number within chromosome
112	4	13
349	11	19
418	13	22
429	13	33
602	19	8

$$v_{1} = (011001111110110101100001101111000)$$
$$v_{2} = (100011000101110000100011111011110)$$
$$v_{3} = (001000100000110101111011011111011)$$
$$v_{4} = (011001111110010101100001101111000)$$
$$v_{5} = (000101010011111111110000110001100)$$
$$v_{6} = (100011000101101001111000001110010)$$
$$v_{7} = (111011101101110000100011111011110)$$
$$v_{8} = (000111011001010011010111111000101)$$
$$v_{9} = (011001111110110101100001101111000)$$
$$v_{10} = (000010000011001000001010111011101)$$
$$v_{11} = (111011101101101001011000001110010)$$
$$v_{12} = (010000000101100010110000001111100)$$
$$v_{13} = (000101000010010101000100001000111)$$
$$v_{14} = (100001100001110100010110101100111)$$
$$v_{15} = (101110010110011110011000101111110)$$
$$v_{16} = (100110100000001111111010011011111)$$
$$v_{17} = (000001111000110000011010000111011)$$
$$v_{18} = (111011111010001000111010111111011)$$
$$v_{19} = (111011100101110000100011111011110)$$
$$v_{20} = (110011110000011111100001101001011)$$

(2nd generation 의 Fitness)

$$eval(\boldsymbol{v}_1) = f(3.130078, 4.996097) = 23.410669$$
$$eval(\boldsymbol{v}_2) = f(5.279042, 5.054515) = 18.201083$$
$$eval(\boldsymbol{v}_3) = f(-0.991471, 5.680258) = 16.020812$$
$$eval(\boldsymbol{v}_4) = f(3.128235, 4.996097) = 23.412613$$
$$eval(\boldsymbol{v}_5) = f(-1.746635, 5.395584) = 20.095903$$
$$eval(\boldsymbol{v}_6) = f(5.278638, 5.593460) = 17.406725$$
$$eval(\boldsymbol{v}_7) = f(11.089025, 5.054515) = 30.060205$$
$$eval(\boldsymbol{v}_8) = f(-1.255173, 4.734458) = 25.341160$$
$$eval(\boldsymbol{v}_9) = f(3.130078, 4.996097) = 23.410669$$
$$eval(\boldsymbol{v}_{10}) = f(-2.516603, 4.390381) = 19.526329$$
$$eval(\boldsymbol{v}_{11}) = f(11.088621, 4.743434) = 33.351874$$
$$eval(\boldsymbol{v}_{12}) = f(0.795406, 5.381472) = 16.127799$$
$$eval(\boldsymbol{v}_{13}) = f(-1.811725, 4.209937) = 22.692462$$
$$eval(\boldsymbol{v}_{14}) = f(4.910618, 4.703018) = 17.959701$$
$$eval(\boldsymbol{v}_{15}) = f(7.935998, 4.757338) = 13.666916$$
$$eval(\boldsymbol{v}_{16}) = f(6.084492, 5.652242) = 26.019600$$
$$eval(\boldsymbol{v}_{17}) = f(-2.554851, 4.793707) = 21.278435$$
$$eval(\boldsymbol{v}_{18}) = f(11.134646, 5.666976) = 27.591064$$
$$eval(\boldsymbol{v}_{19}) = f(11.059532, 5.054515) = 27.608441$$
$$eval(\boldsymbol{v}_{20}) = f(9.211598, 4.993762) = 23.867227$$

Geberation	Total Fitness	Best Fitness
1st	387.776682	30.060205 (v_{15})
2nd	447.049688	33.351874 (v_{11})

(1000th Generation)

Population

$\boldsymbol{v}_1 = (111011110110011011100101010111011)$
$\boldsymbol{v}_2 = (111001100110000100010101010111000)$
$\boldsymbol{v}_3 = (111011110111011011100101010111011)$
$\boldsymbol{v}_4 = (111001100010000110000101010111001)$
$\boldsymbol{v}_5 = (111011110111011011100101010111011)$
$\boldsymbol{v}_6 = (111001100110000100000100010100001)$
$\boldsymbol{v}_7 = (110101100010010010001100010110000)$
$\boldsymbol{v}_8 = (111101100010001010001101010010001)$
$\boldsymbol{v}_9 = (111001100010010010001100010110001)$
$\boldsymbol{v}_{10} = (111011110111011011100101010111011)$
$\boldsymbol{v}_{11} = (110101100000010010001100010110000)$
$\boldsymbol{v}_{12} = (110101100010010010001100010110001)$
$\boldsymbol{v}_{13} = (111011110111011011100101010111011)$
$\boldsymbol{v}_{14} = (111001100110000100000101010111011)$
$\boldsymbol{v}_{15} = (111001101010111001010100110110001)$
$\boldsymbol{v}_{16} = (111001100110000101000100010100001)$
$\boldsymbol{v}_{17} = (111001100110000100000101010111011)$
$\boldsymbol{v}_{18} = (111001100110000100000101010111001)$
$\boldsymbol{v}_{19} = (111101100010001010001110000010001)$
$\boldsymbol{v}_{20} = (111001100110000100000101010111001)$

Fitness

$eval(\boldsymbol{v}_1) = f(11.120940, 5.092514) = 30.298543$
$eval(\boldsymbol{v}_2) = f(10.588756, 4.667358) = 26.869724$
$eval(\boldsymbol{v}_3) = f(11.124627, 5.092514) = 30.316575$
$eval(\boldsymbol{v}_4) = f(10.574125, 4.242410) = 31.933120$
$eval(\boldsymbol{v}_5) = f(11.124627, 5.092514) = 30.316575$
$eval(\boldsymbol{v}_6) = f(10.588756, 4.214603) = 34.356125$
$eval(\boldsymbol{v}_7) = f(9.631066, 4.427881) = 35.458636$
$eval(\boldsymbol{v}_8) = f(11.518106, 4.452835) = 23.309078$
$eval(\boldsymbol{v}_9) = f(10.574816, 4.427933) = 34.393820$
$eval(\boldsymbol{v}_{10}) = f(11.124627, 5.092514) = 30.316575$
$eval(\boldsymbol{v}_{11}) = f(9.623693, 4.427881) = 35.477938$
$eval(\boldsymbol{v}_{12}) = f(9.631066, 4.427933) = 35.456066$
$eval(\boldsymbol{v}_{13}) = f(11.124627, 5.092514) = 30.316575$
$eval(\boldsymbol{v}_{14}) = f(10.588756, 4.242514) = 32.932098$
$eval(\boldsymbol{v}_{15}) = f(10.606555, 4.653714) = 30.746768$
$eval(\boldsymbol{v}_{16}) = f(10.588814, 4.214603) = 34.359545$
$eval(\boldsymbol{v}_{17}) = f(10.588756, 4.242514) = 32.932098$
$eval(\boldsymbol{v}_{18}) = f(10.588756, 4.242410) = 32.956664$
$eval(\boldsymbol{v}_{19}) = f(11.518106, 4.472757) = 19.669670$
$eval(\boldsymbol{v}_{20}) = f(10.588756, 4.242410) = 32.956664$

∗ Best Fitness = 35.477938

※ 주의 사항

다음 Generation 의 Best Fitness 가 반드시 좋은 것은 아니다.

(ex) 396th generation 의 best fitness = 38.827553

(원인) Stochastic Errors of Sampling

(대책) 각 generation 의 best fitness 및 해당 chromosome 저장