최적화 기법

Optimization Technique

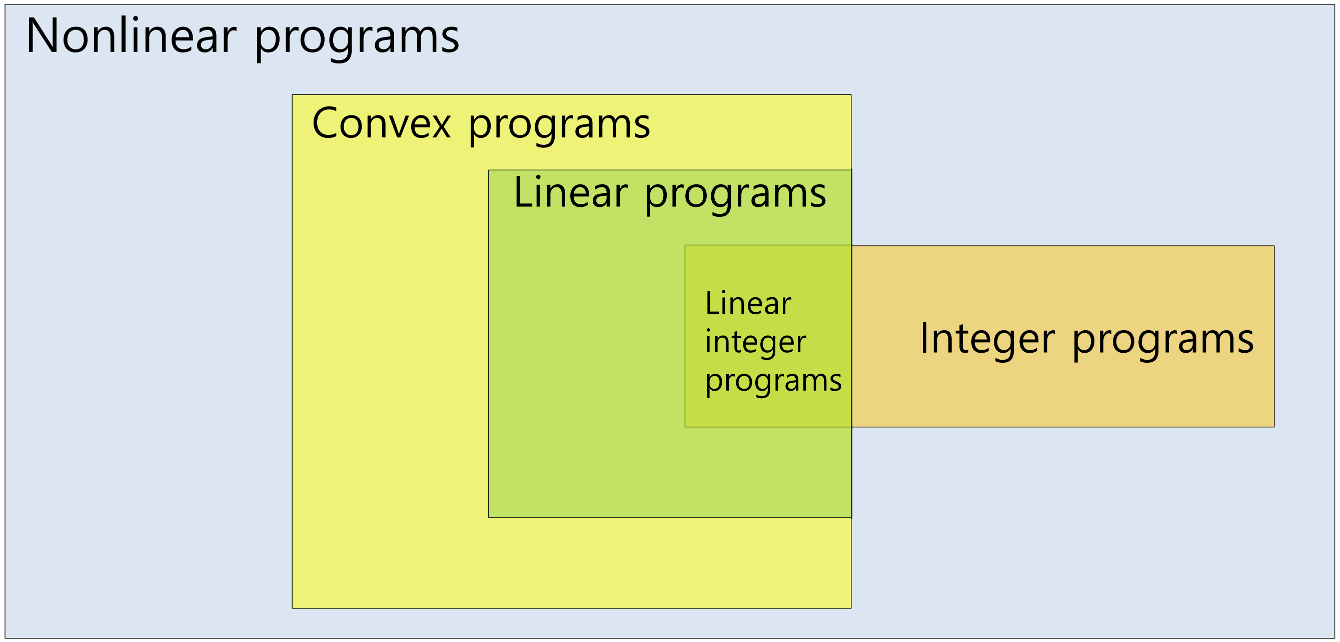
Optimization Problems

- General problem: nonlinear programming

Find x

Minimize	$f(x)$	
Subject to	$g_i(x) \geq 0$	$i = 1, \ldots, m$
	$h_j(x) = 0$	$j = 1, \ldots, p$

f, g_i, h_j : general functions of the parameter $x \in R^n$



Optimization Problems

Programs	f	g_i	h_j	$\boldsymbol{x}$	특징
Nonlinear program	nonlinear	nonlinear	nonlinear	R^n	
Convex program	convex	concave	linear	R^n	
Linear Program	linear	linear	linear	R^n	
Integer Program	nonlinear	nonlinear	nonlinear	I^n	
Linear Integer Program	Linear	linear	linear	I^n	

Optimization Problems

- Car manufacturing problem
  - Car model A, B

Car	Assembly	Finishing	Profit
A	4 h	6 h	400 $
B	6 h	3 h	300 $

  - Supply Constraints

    Available assembly time: 720 h

    Available finishing time: 480 h
  - Demand Constraints

    A : 20 대 이상

    B : 30 대 이상

➢ 최대 profit 을 위한 model A, B 의 생산 대수?

Optimization Problems

- Diet problem

Convex Function

- Convex combination

Given two points $x, y \in R^n$,
a convex combination is any point of the form
$$z = \lambda x + (1-\lambda) y, \qquad \lambda \in R^1 \text{ and } 0 \le \lambda \le 1$$

- Convex set

A set $S \subseteq R^n$ is convex, if it contains all convex combinations of pairs of points $x, y \in R^n$

- Convex function

Let $S \subseteq R^n$ be a convex set.
The function $c : S \rightarrow R^1$ is convex, if for any two points $x, y \in S$,
$$c(\lambda x + (1-\lambda) y) \le \lambda\, c(x) + (1-\lambda)\, c(x)$$

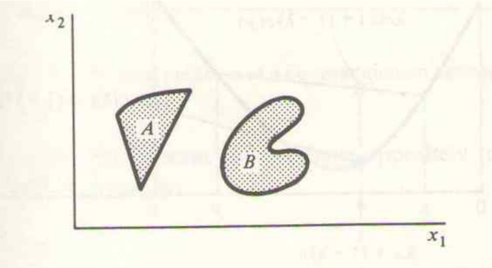


Figure 1–7 A convex set A and a nonconvex set B.

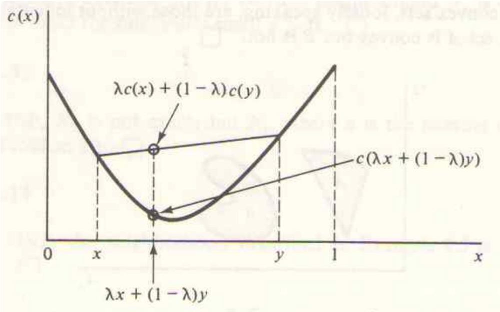


Figure 1–8 A function c convex in [0, 1].

A function f is **concave** iff $-f$ is convex

Linear function is a convex function

Local & Global Optima

- Neighborhood : $N(f)$
  - 현재의 solution f 주변 (이웃) 의 값들
- Local optima

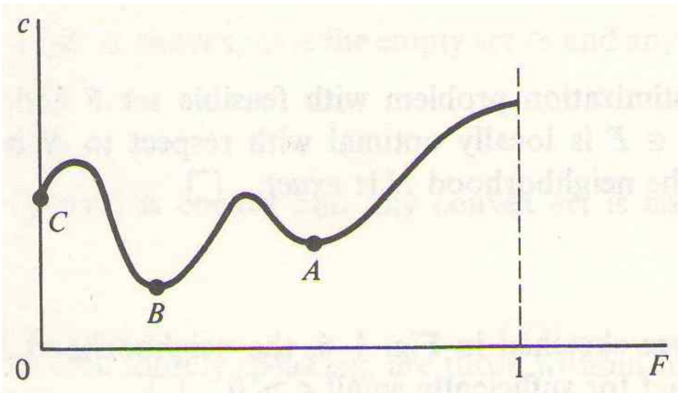
  A feasible solution f is locally optimal if

  $$c(f) \leq c(g) \quad \text{for all} \quad g \in N(f)$$

- Global optimum

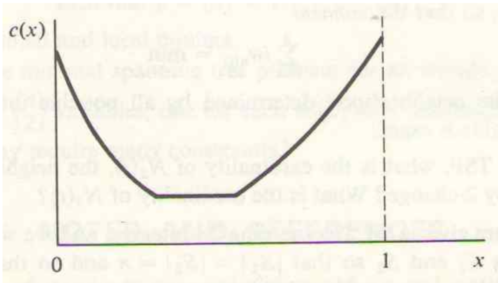
  A feasible solution f is (globally) optimal if

  $$c(f) \leq c(g) \quad \text{for all} \quad g \in S$$



Convex Programming

In a convex programming problem,
every point locally optimal is also globally optimal.



- Linear programming problem
  - Global optimal solution ☞ simplex method

- Linear & Integer programming problem
  - Global optimal solution

(ex) shortest path problem: Dijkstra algorithm
max-flow problem: Ford-Fulkerson algorithm
……

Non-Convex Programming

- Global optimal solution 을 구하는 알고리즘이 없다
  - NP complete

- Near-optimal solution
  - 근사적 최적해 / 합리적 수준의 해
  - Traditional approach
    - Local search
    - Greedy algorithm
    - Dynamic programming
    - Branch & Bound
    - A* algorithm
  - Modern approach
    - Simulated annealing (SA)
    - Tabu search (TS)
    - Genetic algorithm (GA)
    - Ant colony optimization (ACO)

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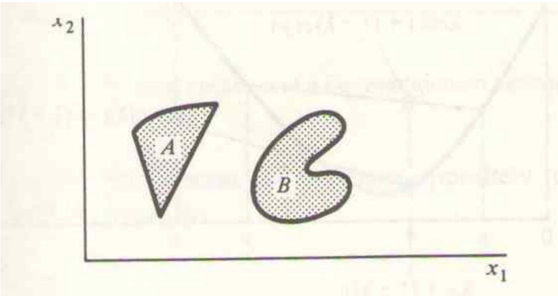

Figure 1–7 A convex set *A* and a nonconvex set *B*.

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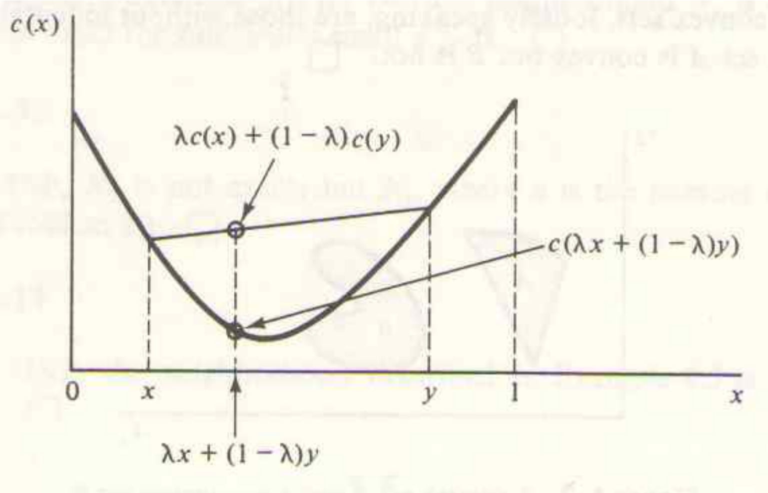

Figure 1–8 A function *c* convex in [0, 1].

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Convex Function

(Q) Prove that $f(x) = x^2 + 2$ is convex function

(Sol) For $x_1, x_2 \in R$

$f(x_1) = x_1^2 + 2, \; f(x_2) = x_2^2 + 2$

$f(\lambda x_1 + (1-\lambda)x_2) = (\lambda x_1 + (1-\lambda)x_2)^2 + 2$

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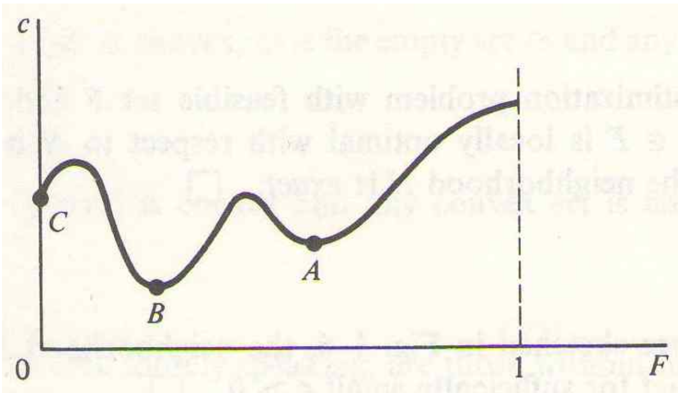

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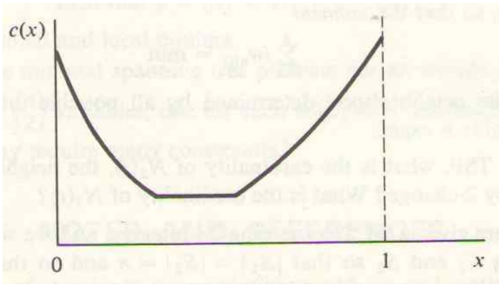

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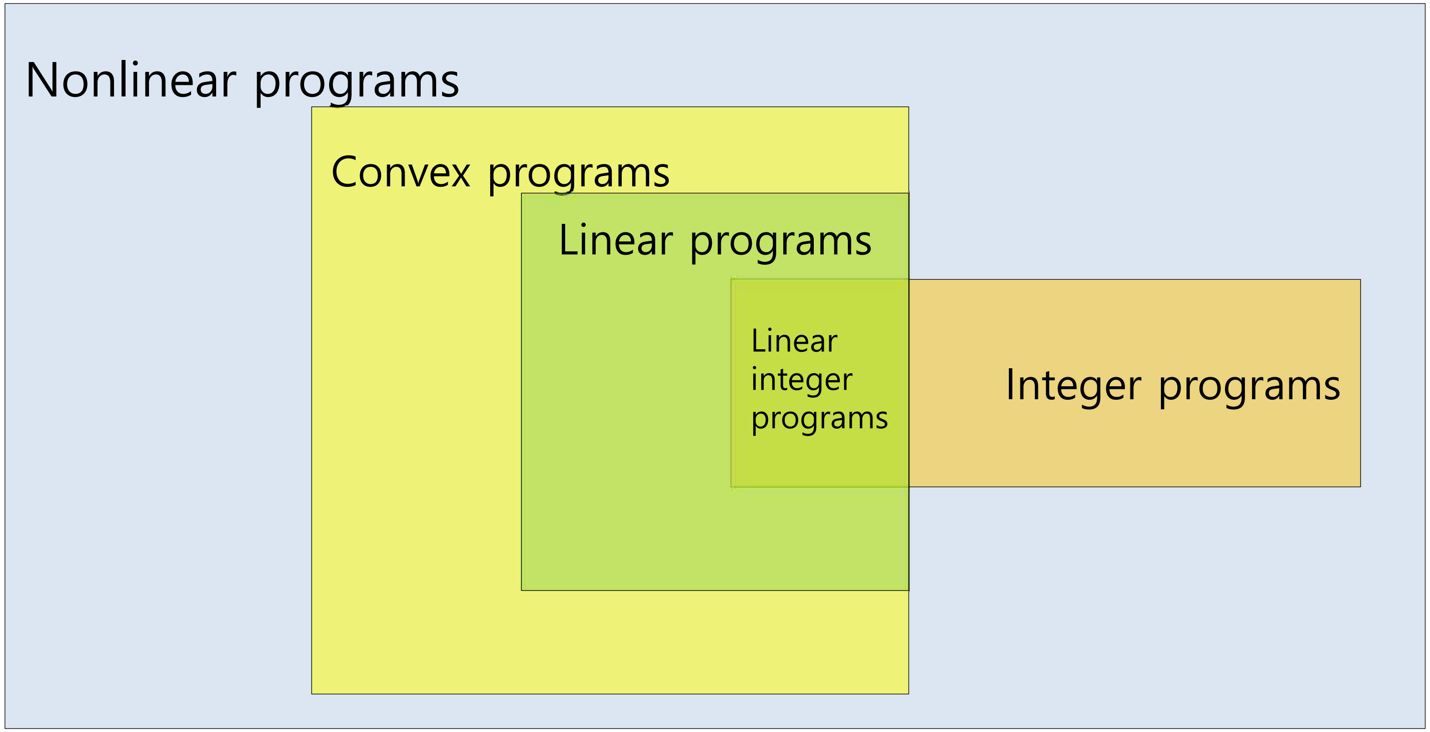
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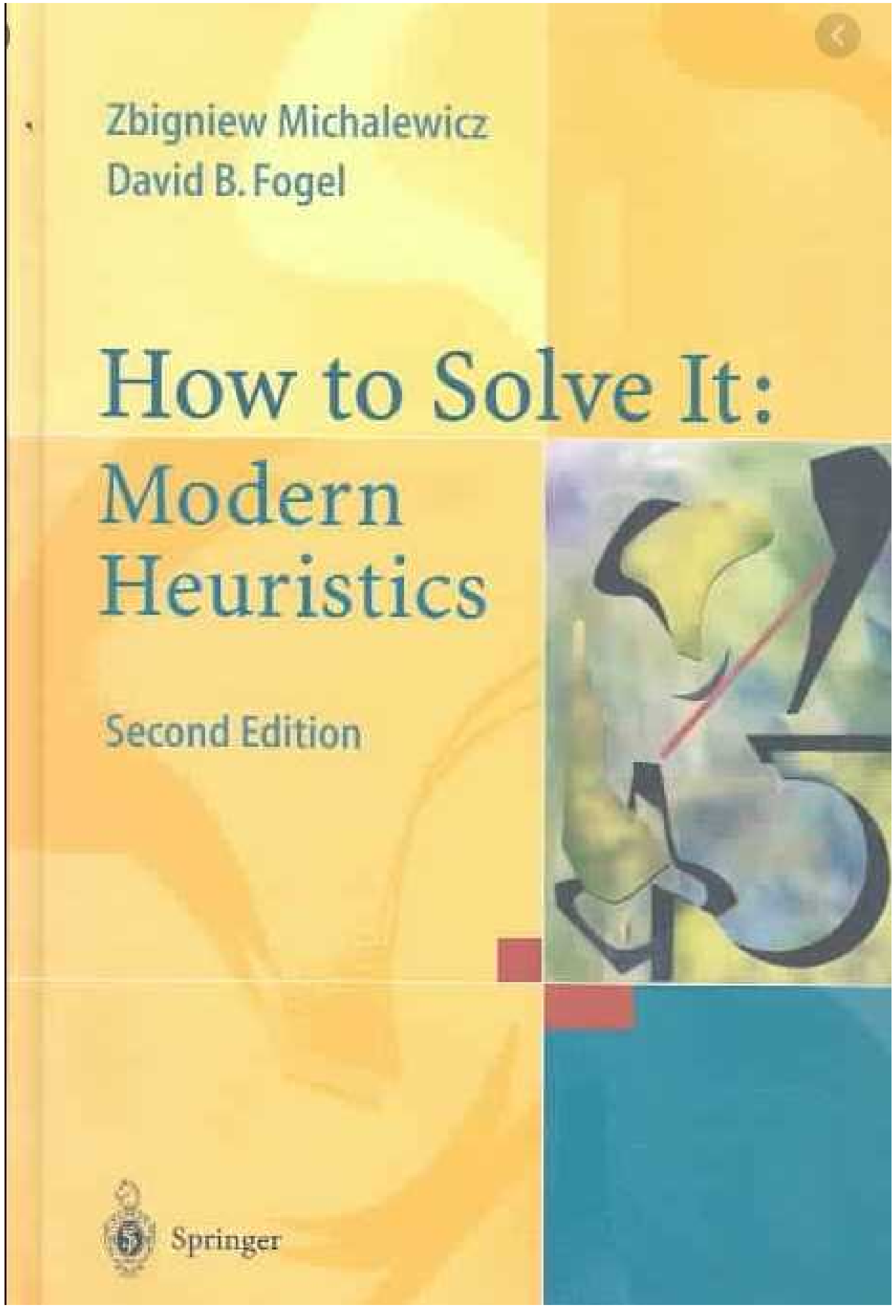
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